International Journal of Advanced Research in Engineering and Technology (IJARET)
Volume 11, Issue 11, November 2020, pp.2382-2386, Article ID: IJARET_11_11_236
Available online at https://iaeme.com/Home/issue/IJARET?Volume=11&Issue=11
ISSN Print: 0976-6480 and ISSN Online: 0976-6499
DOI: https://doi.org/10.34218/IJARET.11.11.2020.236

MINIMAL OF v-OPEN SETS AND v-CLOSED SETS

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ABSTRACT

In this paper a new class of minimal open and minimal closed sets in topological spaces, namely minimal v-open and minimal v-closed sets are introduced. We give some basic properties and various characterizations of minimal v-open and minimal v-closed sets.

Mathematical Subject Classification: 54A05

Key words: minimal v-open, minimal v-closed

Cite this Article: S. Saranya and K. Bageerathi, Minimal of v-Open Sets and v-closed Sets, *International Journal of Advanced Research in Engineering and Technology*, 11(11), 2020, pp. 2382-2386.
https://iaeme.com/Home/issue/IJARET?Volume=11&Issue=11

1. INTRODUCTION

The concepts of minimal open sets in topological spaces were introduced and considered by Nakaoka and Oda in [3] and [4]. More precisely, in 2001, Nakaoka and Oda [4] characterized the notions of minimal open sets and proved that any subset of a minimal open set is pre-open. Also, as an application of a theory of minimal open sets, they obtained a sufficient condition for a locally finite space to be a pre-Hausdorff space

In this paper, the concepts of minimal v-open sets and minimal v-closed sets are introduced. Some basic fundamental properties of minimal v-open sets are given. Also minimal v-closed sets is defined and its properties are investigated. Further, their relationship with already existing concepts in topology are discussed [7].

2. PRELIMINARIES

Definition 2.1.[2] Let (X, τ) be a topological space. A subset A of X is said to be g-closed if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ). The complement of a g-closed set is g-open.

Definition 2.2.[1] If A is a subset of a topological space X, then

- the generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by $cl^*(A)$.
- the generalized interior of A is defined as the union of all g-open sets contained in A and is denoted by $int^*(A)$.

Definition 2.3.[5] A subset A of a topological space (X,τ) is said to be v-open if $A \subseteq int^*(cl(A)) \cup cl^*(int(A))$.The collection of all v-open sets in (X,τ) is denoted by $v\text{-}O(X,\tau)$ or simply by $v\text{-}O(X)$.

Definition 2.4.[6] Let A be a subset of a topological space (X,τ). Then the union of all v-open sets contained in A is called the v-interior of A and it is denoted by $vint(A)$. That is, $vint(A) = \cup\{V{:}V \subseteq A$ and $V \in v\text{-}O(X)\}$.

Lemma 2.5.[6] Let A be a subset of a topological space (X,τ). Then

$vint(A)$ is the largest v-open set contained in A.

A is v-open if and only if $vint(A) = A$.

Lemma 2.6.[5] A subset A of a topological space (X,τ) is said to be v-closed if $int^*(cl(A)) \cap cl^*(int(A)) \subseteq A$.

Lemma 2.7.[5] (i)Arbitrary union of v-open sets is v-open.

(ii)Arbitrary intersection of v-closed sets is v-closed.

Definition 2.8.[6] Let A be a subset of a topological space (X,τ). Then the intersection of all v-closed sets in X containing A is called the v-closure of A and it is denoted by $vcl(A)$. That is, $vcl(A) = \cap\{F{:}\, A \subseteq F$ and $F \in v\text{-C}(X)\}$.

Lemma 2.9[6]. Let A be a subset of a topological space (X,τ). Then

- $vcl(A)$ is the smallest v-closed set containing A.
- A is v-closed if and only if $vcl(A) = A$.

3. MINIMAL v-OPEN SETS

In this section, the minimal v-open set is defined and its properties are studied.

Definition 3.1. A proper non-empty v-open subset U of X is said to be a Minimal v-open set if any v-open set contained in U is ϕ or U.

Example 3.2. Let $X = \{a,b,c\}$ and $\tau = \{\phi,\{b\},\{b,c\},X\}$ here

$v\text{-}O(X,\tau)$=$\{\phi,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X\}$. The minimal v-open sets are $\{b\}$ and $\{c\}$.

Theorem 3.3. Let U be a minimal v-open set and W be a v-open set. Then $U \cap W = \phi$ or $U \subset W$. Let U and V be minimal v-open sets. Then $U \cap V = \phi$ or $U = V$.

Proof. Let U be a minimal v-open set and W be a v-open set. If $U \cap W = \phi$, then there is nothing to prove. If $U \cap W \neq \phi$. Then $U \cap W \subset U$. Since U is a minimal v-open set, $U \cap W = U$. Therefore $U \subset W$. This proves (i).

Let U and V be minimal v-open sets. If $U \cap V \neq \phi$, then $U \subset V$ and $V \subset U$ by (i). Therefore $U = V$. This proves (ii). □

Remark 3.4. Let U be a minimal v-open set. If $x \in U$, then $U \subset W$ for some v-open set W containing x.

Theorem 3.5. Let U be a non-empty v-open set. Then the following three conditions are equivalent.

- U is a minimal v-open set
- $U \subset vcl(S)$ for any non-empty subset S of U

- $vcl(U) = vcl(S)$ for any non-empty subset S of U.

Proof. (i) ⇒ (ii) Let $x \in U$, U be a minimal v-open set and $S(\neq \phi) \subset U$. By Remark 2.4, for any v-open set W containing x, $S \subset U \subset W$ implies $S \subset W$. Now $S = S \cap U \subset S \cap W$. Since $S \neq \phi$, $S \cap W \neq \phi$. Since W is any v-open set containing x, $x \in vcl(S)$. That is $x \in U$ implies $x \in vcl(S)$ implies $U \subset vcl(S)$ for any non-empty subset S of U. This proves (ii).
(ii) ⇒ (iii): Let S be a non-empty subset of U. That is, $S \subset U$ implies $vcl(S) \subset vcl(U)$. Again from (ii), $U \subset vcl(S)$ for any $S(\neq \phi) \subset U$ implies $vcl(U) \subset vcl(vcl(S)) = vcl(S)$. That is, $vcl(U) \subset vcl(S)$. Then $vcl(U) = vcl(S)$ for any non-empty subset S of U. This proves (iii).

(iii) ⇒ (i): From (iii), we have $vcl(U) = vcl(S)$ for any non-empty subset S of U. Suppose U is not a minimal v-open set. Then there exists a non-empty v-open set V such that $V \subset U$ and $V \neq U$. Now there exists an element a in U such that $a \notin V$ implies $a \in X \backslash V$. That is $vcl(\{a\}) \subset vcl(X \backslash V) = X \backslash V$, as $X \backslash V$ is v-closed set in X. It follows that, $vcl(\{a\}) \neq vcl(U)$. This is a contradiction for $vcl(\{a\}) = vcl(U)$ for any $\{a\}(\neq \phi) \subset U$. Therefore U is a minimal v-open set. This proves (i).

Theorem 3.6. Let V be a non-empty finite v-open set. Then there exists atlest one (finite) minimal v-open set U such that $U \subset V$.

Proof. Let V be a non-empty finite v-open set. If V is a minimal v-open set, we may set $U = V$. If V is not a minimal v-open set, then there exists a (finite) v-open set V_1 such that $\phi \neq V_1 \subset V$. If V_1 is a minimal v-open set, we may set $U = V_1$. If V_1 is not a minimal v-open set, then there exists a (finite) v-open set V_2 such that $\phi \neq V_2 \subset V_1$. Continuing this process, we have a sequence of v-open sets $V \supset V_1 \supset V_2 \supset V_3 \supset \ldots\ldots \supset V_k \supset \ldots\ldots$. Since V is a finite set, this process repeats only finitely. Then finally we get a minimal v-open set $U = V_n$ for some positive integer n.

Corollary 3.7. Let X be a locally finite space and V be a non-empty v-open set. Then there exists at least one (finite) minimal v-open set U such that $U \subset V$.

Proof. Let X be a locally finite space and V be a non-empty v-open set. Let x in V. Since X is locally finite space, a finite open set V_x such that x in V_x. Then $V \cap V_x$ is a finite v-open set. By Theorem 2.6, there exists a at least one (finite) minimal v-open set U such that $U \subset V \cap V_x$. That is, $U \subset V \cap V_x \subset V$. Hence there exists at least one (finite) minimal v-open set U such that $U \subset V$.

Corollary 3.8. Let V be a finite minimal open set. Then there exists atleast one (finite) minimal v-open set U such that $U \subset V$.

Proof. Let V be a finite minimal open set. Then by Proposition 2.1.2, V is a non-empty finite v-open set. By Theorem 2.6, there exists a at least one (finite) minimal v-open set U such that $U \subset V$.

4. MINIMAL v-CLOSED SETS

In this section Minimal v-closed sets and Maximal v-open sets in topological spaces are introduced. Further, the characterizations of Minimal v-closed sets and Maximal v-open sets are derived.

Definition 4.1. A proper non-empty v-closed subset F of X is said to be a Minimal v-closed set if any v-closed set contained in F is ϕ or F.

Definition 4.2. A proper non-empty v-open $U \subset X$ is said to be a Maximal v-open set if any v-open set containing U is either X or U.

Proposition 4.3. Let U be a minimal v-closed set and W be a v-closed set. Then $U \cap W = \phi$ or $U \subset W$. Let U and V be minimal v-closed sets. Then $U \cap V = \phi$ or $U = V$.

Proof. Let U be a minimal ν-closed set and W be a ν-closed set. If $U \cap W = \phi$, then there is nothing to prove. If $U \cap W \neq \phi$. Then $U \cap W \subset U$. Since U is a minimal ν-closed set, $U \cap W = U$. Therefore $U \subset W$. This proves (i).

Let U and V be minimal ν-closed sets. If $U \cap V \neq \phi$, then $U \subset V$ and $V \subset U$ by (i). Therefore $U = V$.This proves (ii).

Theorem 4.4. A proper non-empty subset U of X is maximal ν-open set if and only if $X\setminus U$ is a minimal ν-closed set.

Proof. Let U be a maximal ν-open set. Suppose $X\setminus U$ is not a minimal ν-closed set. Then there exists ν-closed set $V \neq X\setminus U$ such that $\phi \neq V \subset X\setminus U$. That is $U \subset X\setminus V$ and $X\setminus V$ is a ν-open set which is a contradiction for U is a minimal ν-closed set.

Conversely let $X\setminus U$ be a minimal ν-closed set. Suppose U is not a maximal ν-open set. Then there exists a ν-open set $E \neq U$ such that $U \subset E \neq X$. That is $\phi \neq X\setminus E \subset X\setminus U$ and $X\setminus E$ is a ν-closed set which is a contradiction for $X\setminus U$ is a minimal ν-closed set. Therefore U is a maximal ν-closed set.

Theorem 4.5. Let U be a non-empty ν-closed set. Then the following three conditions are equivalent.

- U is a minimal ν-closed set
- $U \subset vcl(S)$ for any non-empty subset S of U
- $vcl(U) = vcl(S)$ for any non-empty subset S of U.

Proof. (i) ⇒ (ii): Let $x \in U$, U be minimal ν-closed set and $S(\neq \phi) \subset U$. Then, for any ν-closed set W containing x, $S \subset U \subset W$ implies $S \subset W$. Now $S = S \cap U \subset S \cap W$. Since $S \neq \phi$, $S \cap W \neq \phi$. Since W is any ν-closed set containing x, $x \in vcl(S)$. That is $x \in U$ implies $x \in vcl(S)$ implies $U \subset vcl(S)$ for any non-empty subset S of U. This proves (ii).
(ii) ⇒ (iii): Let S be a non-empty subset of U. That is $S \subset U$ implies $vcl(S) \subset vcl(U)$. Again from (ii) $U \subset vcl(S)$ for any $S(\neq \phi) \subset U$ implies $vcl(U) \subset vcl(vcl(S)) = vcl(S)$. That is, $vcl(U) \subset vcl(S)$. Then we have $vcl(U) = vcl(S)$ for any non-empty subset S of U. This proves (iii).

(iii) ⇒ (i): From (iii), we have $vcl(U) = vcl(S)$ for any non-empty subset S of U. Suppose U is not a minimal ν-closed set. Then there exists a non-empty ν-closed set V such that $V \subset U$ and $V \neq U$. Now there exists an element a in U such that $a \notin V$ implies $a \in X\setminus V$. That is $vcl(\{a\}) \subset vcl(X\setminus V) = X\setminus V$, as $X\setminus V$ is ν-closed set in X. It follows that $vcl(\{a\}) \neq vcl(U)$. This is a contradiction for $vcl(\{a\}) = vcl(U)$ for any $\{a\}(\neq \phi) \subset U$. Therefore U is a minimal ν-closed set. This proves (i).

Theorem 4.6. Let V be a non-empty finite ν-closed set. Then there exists atleast one (finite) minimal ν-closed set U such that $U \subset V$.

Proof. Let V be a non-empty finite ν-closed set. If V is a minimal ν-closed set, we may set $U = V$. If V is not a minimal ν-closed set, then there exists a (finite) ν-closed set V_1 such that $\phi \neq V_1 \subset V$. If V_1 is a minimal ν-closed set, we may set $U = V_1$. If V_1 is not a minimal ν-closed set, then there exists (finite) ν-closed set V_2 such that $\phi \neq V_2 \subset V_1$. Continuing this process, we have a sequence of ν-closed sets $V \supset V_1 \supset V_2 \supset V_3 \supset \ldots\ldots \supset V_k \supset \ldots\ldots$. Since V is a finite set, this process repeats only finitely. Then finally we get a minimal ν-closed set $U = V_n$ for some positive integer n.

Theorem 4.7. A proper non-empty subset F of X is maximal ν-open set if and only if $X\setminus F$ is a minimal ν-closed set.

Proof. Let F be a maximal v-open set. Suppose $X\setminus F$ is not a minimal v-closed set. Then there exists a v-closed set $U \neq X\setminus F$ such that $\phi \neq U \subset X\setminus F$. That is $F \subset X\setminus U$ and $X\setminus U$ is a v-open set which is a contradiction for F is a maximal v-open set.

Conversely, let $X\setminus F$ be a minimal v-closed set. Suppose F is not a maximal v-open set. Then there exists a v-open set $E \neq F$ such that $F \subset E \neq X$. That is $\phi \neq X\setminus E \subset X\setminus F$ and $X\setminus E$ is a v-closed set which is a contradiction for $X\setminus F$ is a minimal v-closed set. Therefore F is a maximal v-open set.

5. CONCLUSION

In this paper, a new class of closed and open sets called minimal(maximal) v-open and minimal(maximal) v-closed sets are defined and their properties are studied.

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